The algorithm of adaptive random search for discrete-continuous optimization[1]

A. Sh. Abakarov, [2] Yu. A. Sushkov [3]
St. Petersburg State University

Abstract

Some optimization approach for solving the mixed discrete-continuous optimization problem is proposed. The application of the proposed approach for structure change systems is presented.

Keywords: discrete-continuous optimization, adaptive random search, change structure systems.

Introduction

The purpose of this paper is to describe the application of adaptive random search [1] for discrete-continuous optimization, in other words, the optimization problem contains both continuous and discrete variables.

There are many diverse applications where the mathematical models are based on discrete-continuous optimization. Note that the optimization of such models in majority cases is difficult because of potential existence of multiple optimum in the domain of the objective function. And so the most general methods for solving discrete-continuous problems are global optimization methods, for example, the genetic algorithms. Here we use the adaptive random search as the optimization algorithm [1]. Note, that for the adaptive random search objective function features are not essential and only the number of the objective function calculation is significant.

We shall begin with the mathematical statement of the discrete-continuous optimization problem.

Let $\Lambda \subset \mathbb{R}^n$ be the continuous set. Denote by S some discrete set. Define the $s \in S$ as a structure. So the discrete-continuous optimization problem can be formulated as follows:

$$F(\lambda, s) \to \min_{\lambda \in \Lambda, s \in S}. \tag{1}$$

Suppose that the constraints for the discrete variables $s \in S$ and continuous variable $\lambda \in \Lambda$ are included into function (1). We can do it because it is not so significant for adaptive random search used.

Suppose furthermore that for any value of continuous variable λ there exists an algorithm for finding the optimal in some sense structure s. In other words for any value $\lambda^* \in \Lambda$ we can find the optimal value $s^* \in S$.

[1]This work was supported by grant 04-01-39002GFEN2004_ a.
[2]e-mail: alik@vega.math.spbu.ru
[3]e-mail: Yu.Sushkov@pobox.spbu.ru

To find the optimum for the continuous variable λ we propose to use the random search and to find the optimal (in some sense) structure s — greedy algorithms, algorithms of dynamic programming and other discrete algorithms.

It is obvious, that problem (1) can be solved by using the combination of random search and some discrete algorithm.

In the next part of this paper we will consider some applications of the described approach for solving some discrete-continuous optimization problem.

Synthesis of a change structure systems

In this section, for the demonstration of the above-mentioned approach application, we will solve the typical problem of synthesis the change structure systems(CSS) [2,3,4].

The change structure systems are ones whose tuning to the definite output function is realizing by the discrete connection change between a functional elements, in other words, by change systems structure.

There are many applications of CSS, for example, multifunctional logical modules, a mechanical box change transmission, change structure microwave devices, electrical filters and other radiotechnical apparatuses.

Further we will briefly describe the two-pole CSS theory [2] and will formulate some synthesis problem. For better description of two-pole CSS theory see work [2].

Note that the above-mentioned approach can also be used for synthesis of three-pole CSS, where the hypergraph theory [2,3,4] is used.

Define the CSS as the set of objects [2]:

$$\Sigma = \langle \Gamma, v, w \rangle, \tag{2}$$

where:

— $\Gamma = \langle Z, D + U \rangle$ is a weighed structural graph of CSS, where:

– Z is a finite set of the graph Γ nodes (or CSS poles) ;

– D is a set of graph Γ arcs (or CSS functional elements);

– U is a set of graph Γ arcs (or CSS control elements for tuning to the definite output function;

— $v \in Z$ is an input node of graph Γ (or input pole of CSS) and $w \in Z$ is an output node of graph Γ (or an output pole of CSS).

Let $\omega_i \in \mathbb{R}$ be signal value on the i-th pole of CSS. Denote by λ_{ij} the parameter of a functional element inlcuding i and j poles, $\lambda_{ij} \in \Lambda \subseteq \mathbb{R}$, $\Lambda = \Lambda_{01} \cup \Lambda_{1\infty}$. Without loss of generality suppose that $\Lambda_{01} \subseteq (0,1)$ and $\Lambda_{1\infty} \subseteq (1,\infty)$. Any element $q \in U$ matches a pair of poles such that by including a control element of U, the signal value on the pole i, i.e. ω_i, becomes equal to signal value on the node j, i.e., ω_j: $\omega_i = \omega_j$.

Let us denote:

— $\Gamma_D = \langle Z_D, D \rangle$, $Z_D = Z$ is a functional subgraph;

— $\Gamma_U = \langle Z_U, U \rangle$ is a control subgraph.

Let $R \subset U$ be a subset such that in the subgraph $\Gamma_{DR} = \langle Z_{DR}, D + R\rangle$ the following unique path exists:

$$i_1, i_2, \ldots, i_n, \quad i_n, \tag{3}$$

where $i_1 = v$, $i_n = w$, $i_j \in Z \;\; n \le d$, $d = |D|$. So path (3) determines some mode of the change structure system Σ for the subset of control elements R.

The signal value for the output pole ω_w for some subset R can be calculated as follows:

$$y_R = \omega_w = \lambda_{i_1 i_2}\lambda_{i_2 i_3} \ldots \lambda_{l_{n-1} l_n}\omega_v, \quad R \subset U,$$

where $\lambda_{i_j i_{j+1}} = 1$ provided that $(i_j, i_{j+1}) \in U$. Thus, the function

$$f_R(\lambda) = \lambda_{i_1 i_2}\lambda_{i_2 i_3}\lambda_{i_{n-1} i_n}$$

is an output function for mode R.

Let $G = \langle g_1, g_2, \ldots, g_l\rangle$ be a given sequence of numbers in $\mathbb{R}$, called an output gamma and $g_i \in G_i = [gl_i, gr_i]$, $i \in 1 : l$, where gl_i and gr_i are left and right bounds of g_i.

Let for any $i \in 1 : l$ exist R_i such that the value of the output function $f_{R_i}(\lambda^*) \in G_i$, $\lambda^* \in \Lambda^l$.

Suppose that the function

$$F_g(\lambda) = \max_{i \in 1:l} \left| \frac{f_{R_i} - g_i}{g_i} \right| \to \min_{\lambda}, \tag{4}$$

is the performance criterion of realization of the given output gamma G.

Let all functional elements of Σ be renumerated from 1 to d. Let us construct the following function:

$$\Theta(\lambda_j) = \begin{cases} \lambda_j, & 0 < \lambda_j < 1, \\ 1/\lambda_j, & 1 < \lambda_j < \infty. \end{cases} \tag{5}$$

Let

$$\Lambda_{01} = [\lambda l, \lambda r], \quad \Lambda_{1\infty} = [1/\lambda r, 1/\lambda l].$$

Let us construct the penalty function

$$\Phi_\lambda = A \sum_{j=1}^{d} \Phi_{\lambda_j}, \tag{6}$$

where A is a sufficiently large number and

$$\Phi_{\lambda_j} = \Big|\lambda l - \Theta(\lambda_j)\Big| + \Big|\lambda r - \Theta(\lambda_j)\Big| + (\lambda l - \lambda r). \tag{7}$$

Thus, the problem of synthesis under consideration may be stated as follows.
Given:
– the gamma $G = \langle g_1, g_2, \ldots, g_l\rangle$,
– the domain $\Lambda = [\lambda l, \lambda r] \cup [1/\lambda r, 1/\lambda l]$,

– the intervals $\{[gl_i, gr_i] | i \in 1 : l\}$.
Find:
– the functional graph $\Gamma_D = \langle Z_D, D\rangle$,
– the values of functional elements λ_{ij}, such that:
  – $gl_i \le y_i(\lambda) \le gr_i$, $i \in 1 : l$,
  – $\lambda_{ij} \in \Lambda$, $(i, j) \in D$,
  – the criterion (4) $F_g(\lambda)$ has minimal value.

Note, that to describe the CSS theory and to formulate some synthesis problem we have done some simplifications and these simplifications are not essential in our case.

The algorithm for a stated synthesis problem

Step 1. By using the adaptive random search algorithm the values of vector $\lambda^* \in \Lambda$: $\lambda_{12} < \lambda_{13} < \ldots < \lambda_{1,z-1}$ are randomly selected (i.e the optimization for a continuous variable is being done).

Step 2. By using the formula

$$\lambda_{ij} = \lambda_{i1}\lambda_{1j} = \frac{\lambda_{1j}}{\lambda_{1i}}, \quad i, j \in 2 : (z-1),$$

for all arcs of complete graph Γ_z on $|Z|$ nodes, the values of functional elements of are calculated.

Step 3. By using the penalty function (7) and the vector λ^* all arcs of complete graph on $|Z|$ nodes are weighed.

Step 4. By using any greedy algorithm for minimal spanning tree (MST) finding (i.e. the algorithm of Kruskals or the Prim's algorithm) the MST T of graph Γ_z is formed (i.e the optimization for discrete variable is being done).

Step 5. The value of the objective function

$$F(\lambda) = F_g(\lambda) + \Phi_\lambda$$

for the vector λ^* is calculated (this value is necessary need for the adaptive random search).

Step 6. Algorithm is completed if the current iteration of the random search is equal to *nstep*, where *nstep* is the parameter of random search

Algorithm application results

Let us apply the described algorithm to the solution of the following CSS synthesis problem [5].

<u>Synthesis problem</u>.

Given:

– the output gamma: $G = \langle 0.30, 0.52, 0.65, 0.85, 1.05, 2.10, 3.00, 4.00, 4.60, 5.00\rangle$,
– left and right bounds $[gl_i, gr_i]$ for g_i:

[0.25,0.35],[0.40,0.58],[0.60,0.70],[0.80,0.90],[1.00,1.10],
[1.80,2.50],[2.80,3.20],[3.90,4.10],[4.50,4.70],[4.90,5.10];

– the domain of functional elements parameter: $\Lambda = [0.4166, 0.833] \cup [1.2, 2.40]$.
The obtained values of output functions:

$$\begin{array}{ll} \lambda_{1\,11} = 0.299, & \lambda_{12} = 0.519, \\ \lambda_{13} = 0.649, & \lambda_{14} = 0.849, \\ \lambda_{15} = 1.049, & \lambda_{16} = 2.100, \\ \lambda_{17} = 3.005, & \lambda_{18} = 3.948, \\ \lambda_{19} = 4.588, & \lambda_{1\,10} = 4.953. \end{array}$$

The values of the functional elements parameter:

$$\begin{array}{ll} \lambda_{21} = 1.924, & \lambda_{52} = 2.017, \\ \lambda_{65} = 2.003, & \lambda_{86} = 1.879, \\ \lambda_{11\,2} = 1.733, & \lambda_{31} = 1.541, \\ \lambda_{42} = 1.635, & \lambda_{96} = 2.184, \\ \lambda_{79} = 1.527, & \lambda_{10\,7} = 1.648, \end{array}$$

The obtained functional graph of the synthesis problem is shown on the fig. 1.

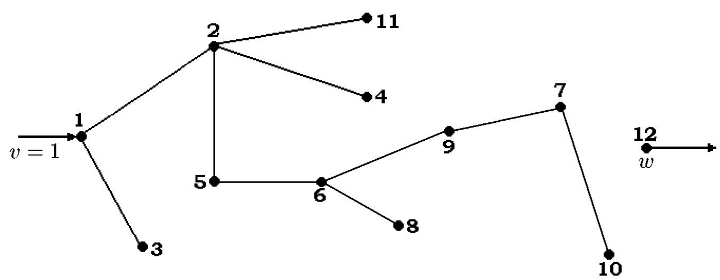

Figure 1: The given functional graph of the synthesis problem.

The time for the synthesis problem solution by using the computer with CPU Pentium-III is equal to 0.3 seconds.

Another way to solve the above-mentioned synthesis problem could contain the following stages:

1. All spanning trees of the complete graph Γ_z are formed ("discrete stage").
2. For every spanning tree the optimal vector λ is found ("continuous stage").

Thus it is obvious that our algorithm of mixed discrete-continuous optimization is more effective because the discrete and continuous stages of the synthesis problem are simultaneously solved. We don't need to form all spanning trees of graph Γ_z. Note the using of adaptive random search in this case is very important.

Conclusion

In future we plan to use the approach for solving some synthesis problems of three-pole CSS, where the hypergraph theory [2,3,4] is being used. We are also going to find another application for a proposed approach.

References

1. Sushkov Yu.A. About one way of the random search organizations // Operation research and statistical modelling. – L., LGU. 1972. – P. 180-185.

2. Sushkov Yu.A. Change structure systems. The abstract of a doctor dissertation. 2000. – SPb., SPbGU. 2000. – 30 p.

3. Sushkov Yu.A. The hypergraphs and matroids connectivity. // Operations research and statistical modelling. – SPb, SPbGU. 1994. Vol. 6. – P. 111-137.

4. Sushkov Yu.A. The connectivity of hypergraphs. – SPb., SPbGU. 2002. – 56 p.

5. Abakarov A.Sh., Sushkov Yu.A. About the synthesis of change structure systems. // Mathematical models. Theory and application. 2003. – SPb., SPbGU. 2003. – P. 3-23.